LOYOLA UNIVERSITY CHICAGO

Department of Mathematics and Statistics
Lake shore campus | 1032 W. Sheridan Road | Chicago, Illinois 60660
Phone 773.508.3558 | Fax 773.508.2123 | www.math.luc.edu

Math 110 - Fall 2024 - Common Final Exam, version A

Print name:_____

Section number:_____ **Instructor's name:**_____

Directions:

- This exam has 13 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to two decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7
Points:	8	10	5	8	6	9	6
Score:							

Question:	8	9	10	11	12	13	Total
Points:	10	10	5	10	7	6	100
Score:							

1. (8 points) A museum charges a flat fee of \$40 for a group of 15 or fewer people. A group of more than 15 people must, in addition to the flat fee of \$40, pay \$2 per person. For example, a group of 16 pays \$72 and a group of 20 pays \$80. The maximum group size is 45.

   (a) Find a piece-wise formula for the cost, C, of having a group of x people in it

   (b) What is the domain of this function?

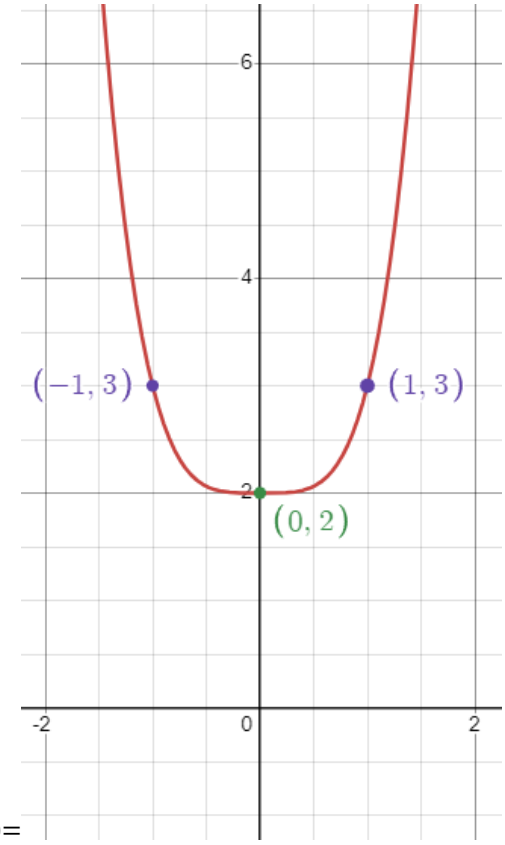
   (c) The museum decides that it must raise the price to \$4 per person for the number of people above 15. Does this affect the domain and/or the range of this function? If so, how are they affected? If not, why are they not affected?

2. (10 points) Consider the function $f(x) = 2x^2 - 18x + 36$

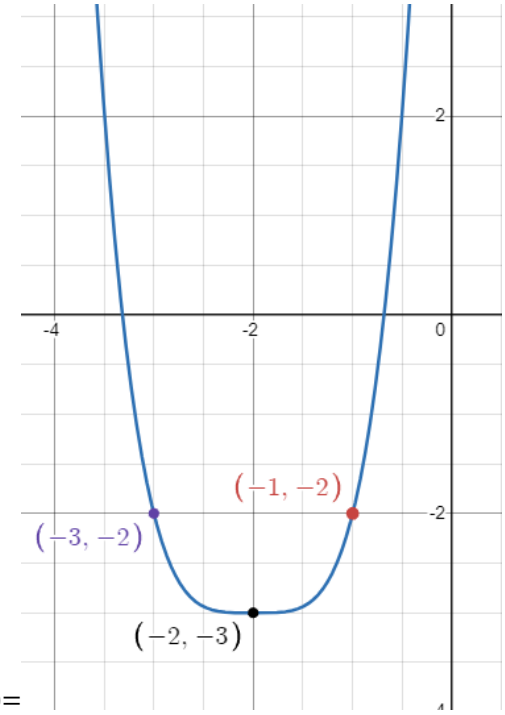
   (a) Is $f(x)$ concave up or concave down?

   (b) Find the zeros of $f(x)$

3. (5 points) The (partial) graphs of $f(x)$ and $g(x)$ are shown below. Find a formula for $g(x)$ in terms of $f(x)$. Your answer should be of the form $g(x) = f(x-h)+k$ for appropriate constants h and k



f(x)=



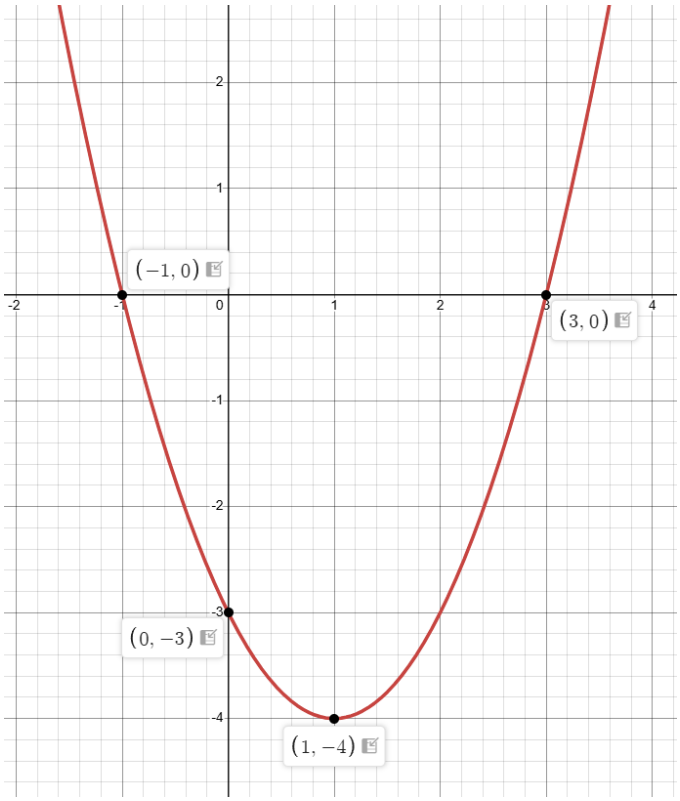
g(x)=

4. (8 points) In a microwave oven, cooking time t is inversely proportional to the square root of the amount of power used w. It takes 8 minutes to heat a frozen dinner at 625 watts.

   (a) Write a formula for the cooking time, t, as a function of power level, w.

   (b) Find the cooking time needed to heat the frozen dinner at the power level $w = 324$ watts.

5. (6 points) Find the formula for the quadratic function $f(x)$ graphed below



6. (9 points) The revenue of Madeline's tutoring business is $\$10,000$ in the year 2024. Recall that a linear function has a general form of $P = mt + b$ and an exponential function has a general form of $P = a \cdot b^t$.

   (a) If the revenue of Madeline's business is decreasing by \$300 per year, find a formula for the function $R(t)$, the revenue t years after 2024.

   (b) If Madeline's revenue is instead increasing by 12% per year, find a formula for the function $R(t)$, the revenue t years after 2024.

   (c) Using your formula from part b, find the year that Madeline's revenue will hit $30,000$. Round to the nearest whole number.

7. (6 points) The profit of Aadi's sandwich shop is increasing at a rate of 7% per year. Find the doubling time (in years).

8. (10 points) Determine if the following functions are linear, exponential, or neither. Then, find an equation for each function or explain why it cannot be done.

(a)

x	-1	0	1	2	3
$f(x)$	-7	-5	-3	-1	1

(b)

x	0	1	2	3	4
$g(x)$	3	6	12	24	48

9. (10 points) Elliot opens a bank account with an initial deposit of $\$20,000$. It earns interest at a nominal rate of 3% per year.

   (a) Find the balance of their account after 5 years if interest is compounded as follows.

   (i) Annually (once per year).

   (ii) Quarterly (4 times per year).

   (iii) Continuously.

   (b) Of the three accounts listed in part (a) of this question, which has the highest effective rate? That is, which account increases the most over the course of one year?

10. (5 points) Let $f(x) = 2x - 5$, $g(x) = -4x + 3$ and $h(x) = e^x$. Find the following, and simplify your answers completely:

   (a) $f(g(2))$

   (b) $h(g(x))$

11. (10 points) Consider the exponential function $Q = 182(0.97)^t$.

(a) Determine if this function displays exponential growth or decay.
Circle one: **exponential growth** or **exponential decay**. Explain your answer in a sentence.

(b) Give the initial value for this function.

(c) Give the growth rate for this function. Write your answer as a percentage.

(d) Give the domain for the given function.

(e) Give the range for the given function.

(f) Write the given function in the form $Q = ae^{kt}$.

12. (7 points) Let $P = f(t) = 700(1.06)^t$ be the population of a town. Let t be measured in years since 2024.

    (a) Evaluate $f(8)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

    (b) Find a formula for $f^{-1}(P)$ in terms of P. Give an exact answer.

    (c) Evaluate $f^{-1}(1000)$. Round to the nearest whole number

    (d) Describe in words what the quantity you found in part c) represents. Write your answer in a complete sentence with units.

13. (6 points) Calculate the following limits

    (a) $\lim_{x \to \infty} \frac{1}{x^3}$

    (b) $\lim_{x \to -\infty} (7x^2 - 9x^3)$

Average rate of change: $\frac{f(b)-f(a)}{b-a}$

Slope-intercept form: $y = b + mx$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Quadratic function: $y = ax^2 + bx + c$

Factored form: $y = a(x - r)(x - s)$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Power function: $y = kx^p$

Directly proportional: $y = kx$

Inversely proportional: $y = \frac{k}{x}$

Factored form of a polynomial: $p(x) = c(x - a_1)(x - a_2) \cdots (x - a_n)$

Exponential Function: $Q(t) = a \cdot b^t$

Continuous Exponential Function: $Q(t) = a \cdot e^{kt}$

Simple Interest: $B = P(1 + r)^t$

Compound Interest: $B = P\left(1 + \frac{r}{n}\right)^{nt}$